Consensus Beyond Thresholds: Generalized Byzantine Quorums Made Live

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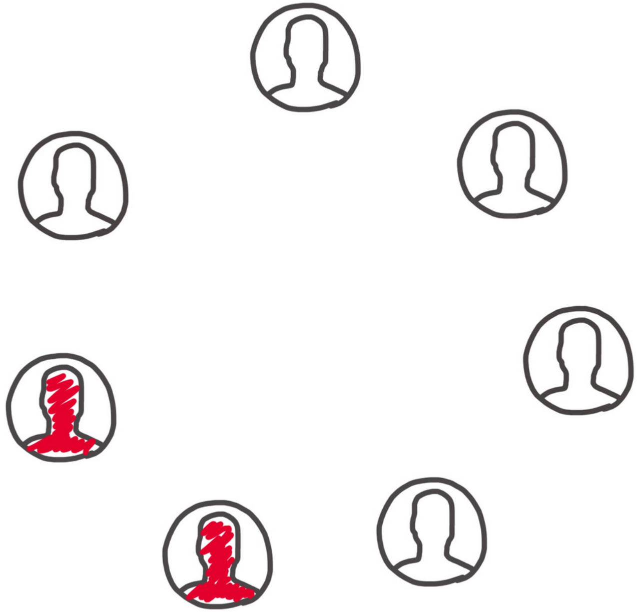
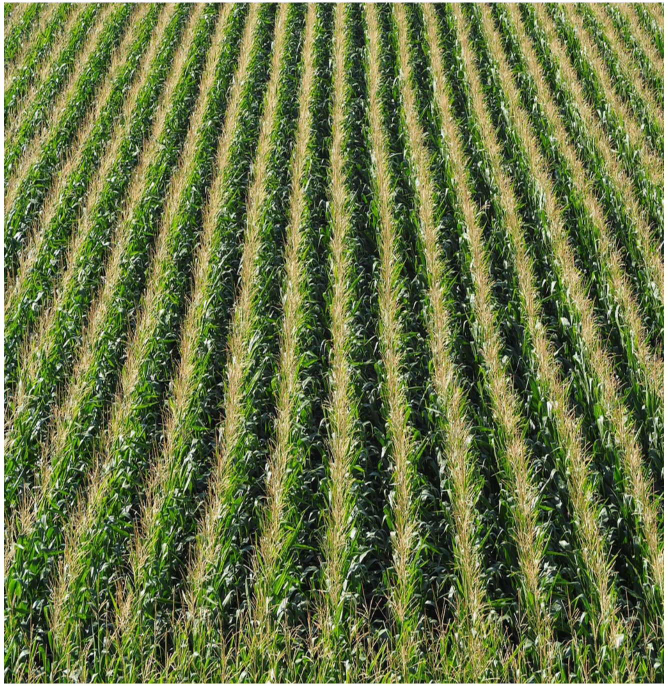
Consensus

- Agree on a common value
- Trust assumptions in thresholds
- $n = 7$
- $f = 2$
- All participants trusted equally



Consensus algorithms still in monoculture

- Agree on a common value
- Trust assumptions in thresholds
- $n = 7$
- $f = 2$
- All participants trusted equally
- Participants are of the same type



The participants are diverse

- Operating system
- Hardware
- Administrators
- Location

- Fail with different probabilities

- Failures are correlated

- Expressive and resilient through complex and correlated trust assumptions

Byzantine quorum systems
A rich and expressive abstraction

Malkhi & Reiter, 1998 [MR98]

- *n* parties: $\mathcal{P} = \{p_1, \ldots, p_n\}$
- Fail-prone system $\mathcal{F}$: A fail-prone set in $\mathcal{F}$ contains all the failed parties
- Quorum System $\mathcal{Q}$: The set of *quorums*
- Such that the *Consistency* and *Availability* conditions hold:

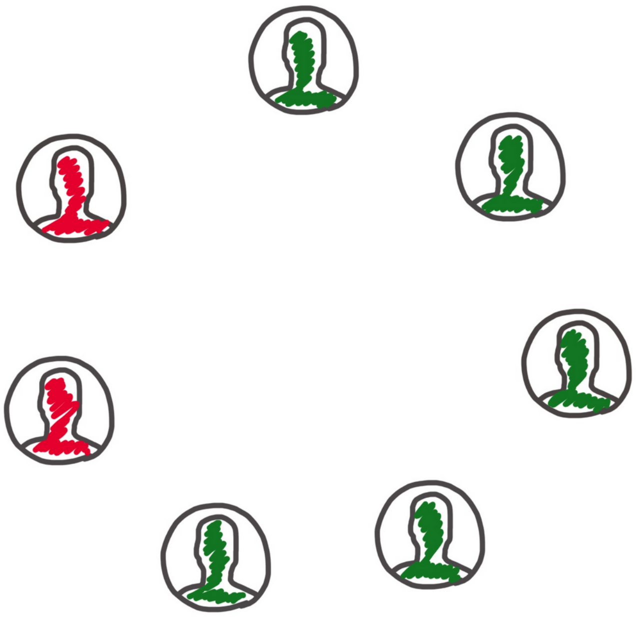
$$\forall Q_1, Q_2 \in \mathcal{Q}, \forall F \in \mathcal{F} : Q_1 \cap Q_2 \not\subseteq F$$

$$\forall F \in \mathcal{F} : \exists Q \in \mathcal{Q} : F \cap Q = \emptyset$$

- By definition generalized

A threshold Byzantine quorum system

- $\mathcal{P} = \{p_1, \ldots, p_7\}$
- $\mathcal{F}$: All subsets with cardinality $f = 2$
- $\mathcal{Q}$: All subsets with cardinality $n - f = 5$
- Consistency
  $|Q_1 \cap Q_2| \geq 3, \; |F| = 2$
- Availability
  Any *2* fail, the other *5* are correct
- $n > 3f$



We need to do better

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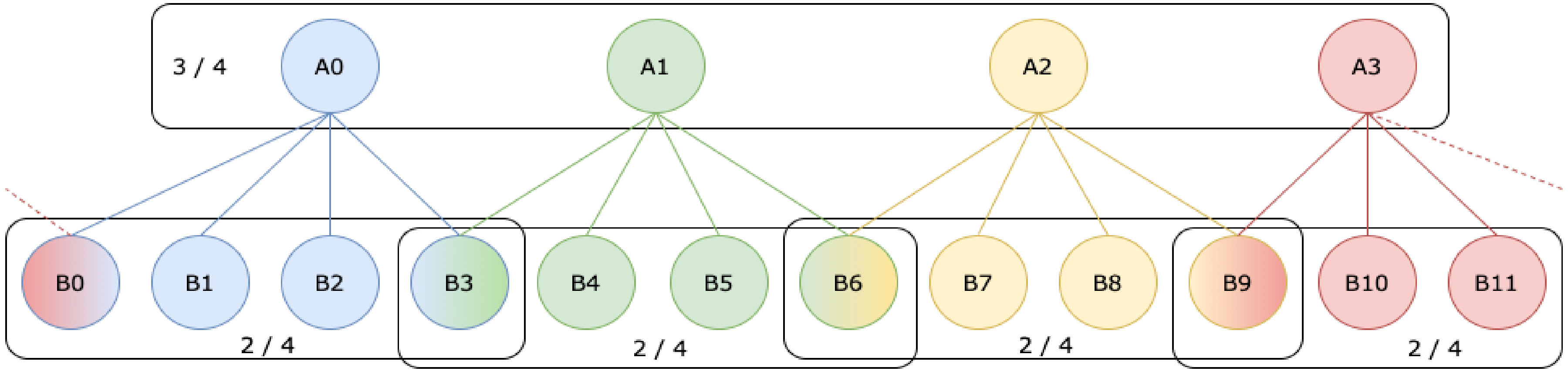
- Generalized BQS → realistic, better resilience, but not yet practical
- **Example**: The 2-layered-1-common generalized BQS



Related work

- Stellar consensus protocol https://www.stellar.org/papers/stellar-consensus-protocol
  - Generalized trust assumptions
  - Different for each user
  - Not based on the classical Byzantine quorum system theory

- Benaloh and Leichter [BL88] first secret sharing over generalized structures

- Hirt and Maurer [HM00] multiparty computation with generalized failure patterns

- Cramer, Damgård, and Maurer [CDM00] use monotone span programs for generalized multiparty computation

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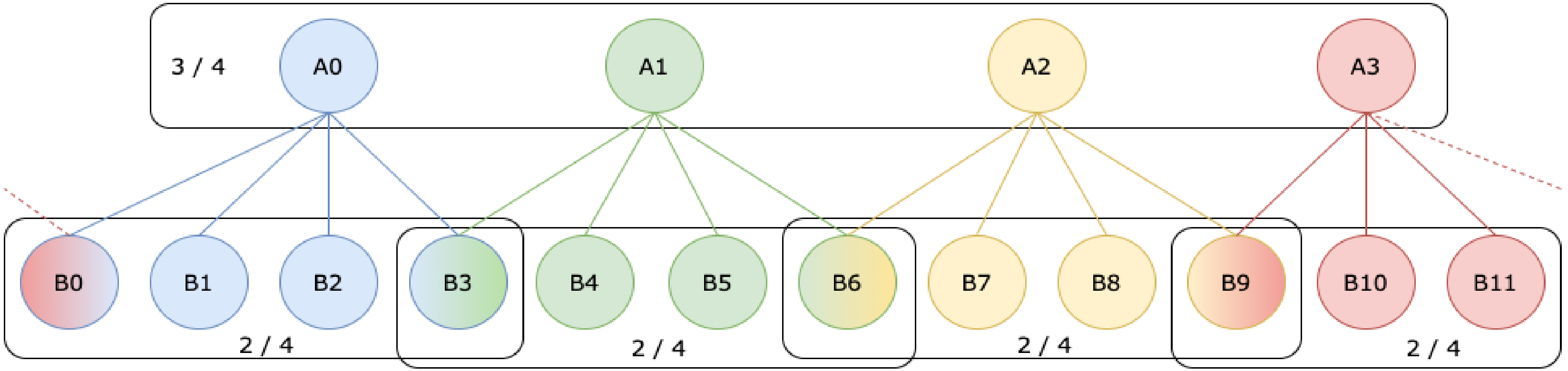
Implementing generalized Byzantine quorum systems

- Challenges (and a solution that would not work).
- Generalized BQS as monotone boolean formulas.
- Generalized BQS as monotone span programs.

Implementing a generalized BQS is a challenging task

Implement BQS as enumeration of all quorums 792 quorums



- Specify in user-friendly way
- Efficient and compact encoding
- Efficient quorum-checking

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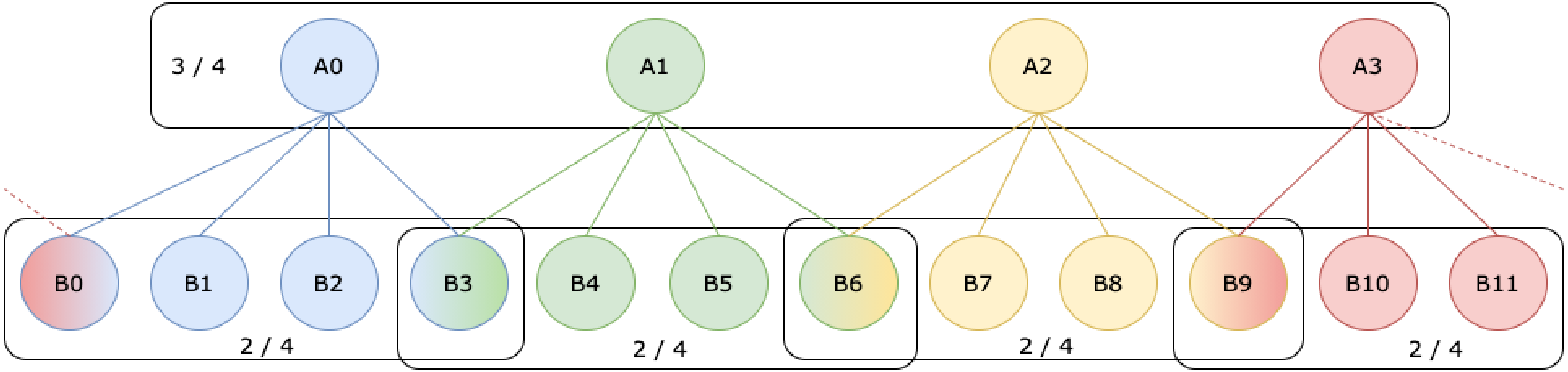
Generalized Byzantine quorum systems as monotone boolean formulas (MBF)

Parsing a BQS as an MBF

- Using logical *and*, *or*, *threshold* operators $\Theta_k^m(q_1, \ldots, q_m)$




```
{"select": 3, "out-of": [
    {"select": 2, "out-of":["A0", {"select": 2, "out-of":["B0", "B1", "B2", "B3"]} ]},
    {"select": 2, "out-of":["A1", {"select": 2, "out-of":["B3", "B4", "B5", "B6"]} ]},
    {"select": 2, "out-of":["A2", {"select": 2, "out-of":["B6", "B7", "B8", "B9"]} ]},
    {"select": 2, "out-of":["A3", {"select": 2, "out-of":["B9", "B10", "B11", "B1"]} ]}
]}
```

Storing the BQS as an MBF

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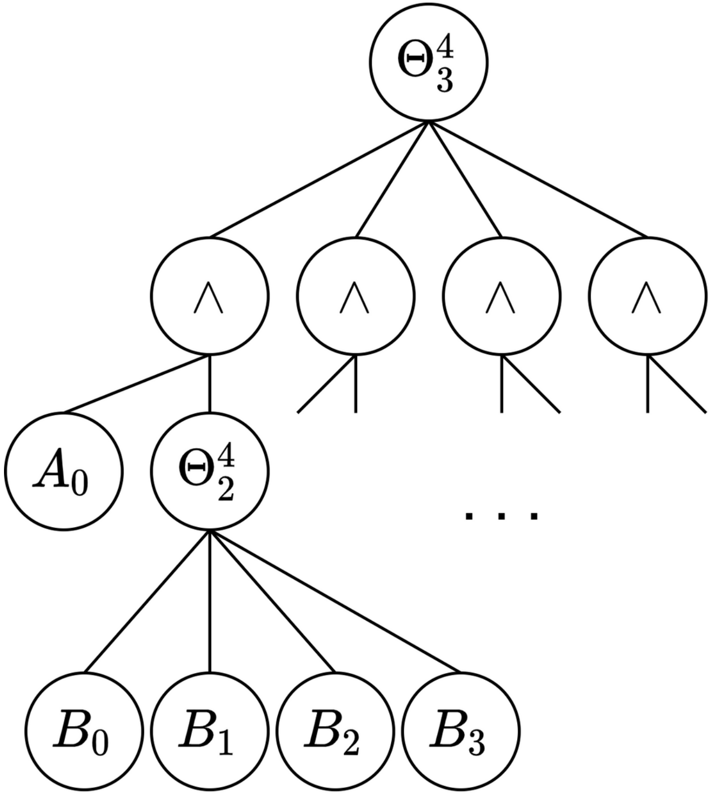
- As a tree
- size is $O(n)$, where n the size of MBF



Checking for quorums

- Check whether set *A* is a quorum
- evaluate formula on input *A*, time *O(n)*

eval(F, A)
    if F is a literal **then**
        return $(F \in A)$
    else
        write $F = op(F_1, \ldots, F_m)$, where $op \in \{\wedge, \vee, \Theta\}$
        for each F_i **do**
            $x_i \leftarrow$ **eval**(F_i, A)
        return $op(x_1, \ldots, x_m)$

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Generalized Byzantine quorum systems as monotone span programs (MSP)

Monotone span programs (MSP)

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- Each participant gets one vector (or more)
- If the vectors of a set of participants span a target vector, the set is accepted
- *An MSP* implements a quorum system if it accepts exactly its quorums
- There are functions efficiently encoded by an MSP, but not by a formula [BGW99]

Parsing a BQS as an MSP

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- Insertion: $\mathcal{Q}_3 = \mathcal{Q}_1(p_z \to \mathcal{Q}_2)$ Nikov, Nikova [NN2004]
  - $\mathcal{Q}_1$ defined on $\mathcal{P}_1$, $p_z \in \mathcal{P}_1$
  - $\mathcal{Q}_2$ defined on $\mathcal{P}_2$
  - $\mathcal{Q}_3$ replaces p_z by quorums in $\mathcal{Q}_2$
- Insertion on MSPs: $\mathcal{M}_3 = \mathcal{M}_1(r_z \to \mathcal{M}_2)$
  - $\mathcal{M}_1$ implements $\mathcal{Q}_1$
  - $\mathcal{M}_2$ implements $\mathcal{Q}_2$
  - $\mathcal{M}_3$ can be constructed to implement $\mathcal{Q}_3$
- Given a formula, create the MSP with recursive insertions of nested sub-formulas

Parsing a BQS as an MSP

- Construct the MSP that implements a given MBF
  - Recursive insertions.
  - The Vandermonde matrix *V(n,t)*, when seen as an MSP, implements the access structure $\Theta_t^n(q_1, \ldots, q_n)$

$$V(n,t) = \begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{t-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{t-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{t-1} \end{pmatrix}$$

$$x_i \neq x_j \neq 0, \text{ for } 1 \leq i \leq j \leq n$$

Checking for quorums using an MSP

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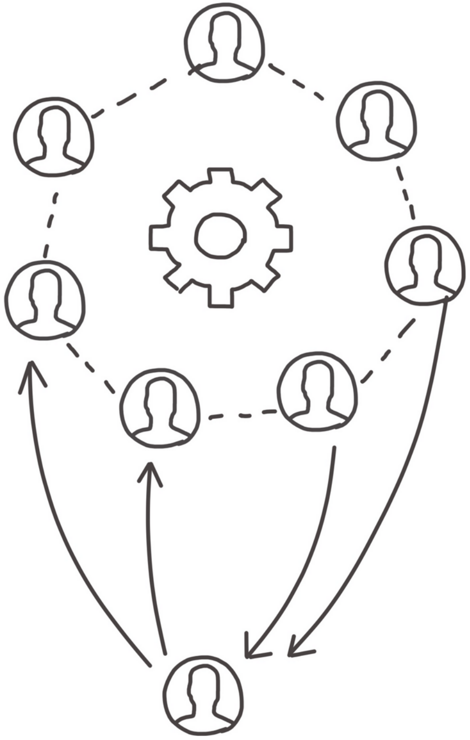
- Check whether $M_A^T \boldsymbol{x} = \boldsymbol{e}_1$ has solutions, using Gaussian elimination.

- Time complexity is $O(n^3)$, where n the dimension of M, can be optimized using PLU-decomposition (but still cubic on average).

Consensus beyond thresholds:
Generalized HotStuff

HotStuff

- Consensus algorithm by Yin *et al* [YMRGA19]
- Efficient, linear communication, speed of network
- Libra cryptocurrency
- Replicas run the protocol
- Clients submit commands and collect responses



Generalized HotStuff

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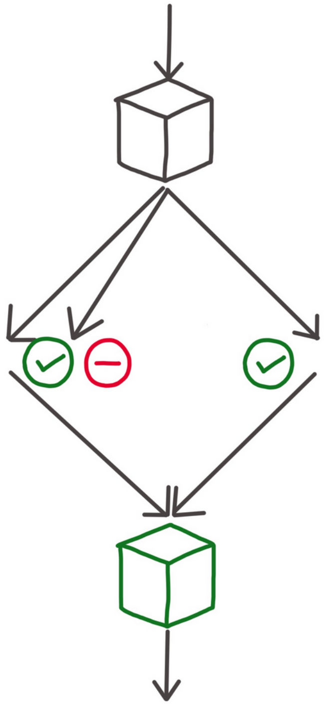
- Protocol advances in epochs

- Each epoch four phases

- In each phase
  - The leader creates a proposal and sends to other replicas
  - The replicas validate and vote
  - The leader waits for ~~$n - f$~~ *a quorum of* votes
  - Upon receiving them, creates a certificate, used in next proposal

- The generalized protocol satisfies the same safety and liveness properties as threshold HotStuff



Evaluated systems

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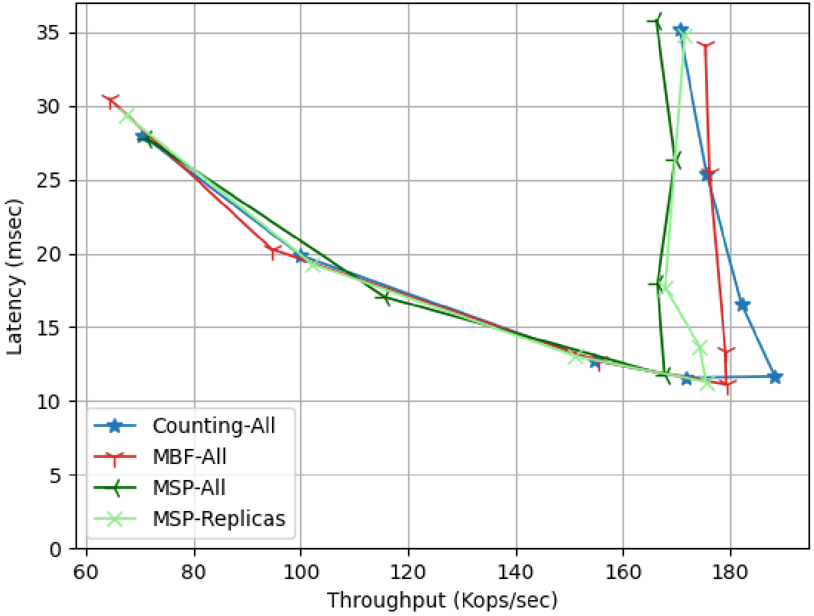
System	BQS implementation in		Supported types of BQS
	replicas	clients	
Counting-All	counting	counting	threshold
MBF-All	MBF	MBF	threshold & generalized
MSP-All	MSP	MSP	threshold & generalized
MSP-Replicas	MSP	-	threshold & generalized

- Based on the *prototype HotStuff* implementation: github.com/hot-stuff/libhotstuff

When the number of parties is small all the generalized protocols are efficient

- 4 replicas, varying number of clients (1 up to 8) and request rate
- All systems instantiated with a threshold BQS with *n = 4, f = 1*



In larger systems the MBF-All protocol is as efficient as the original Counting-All

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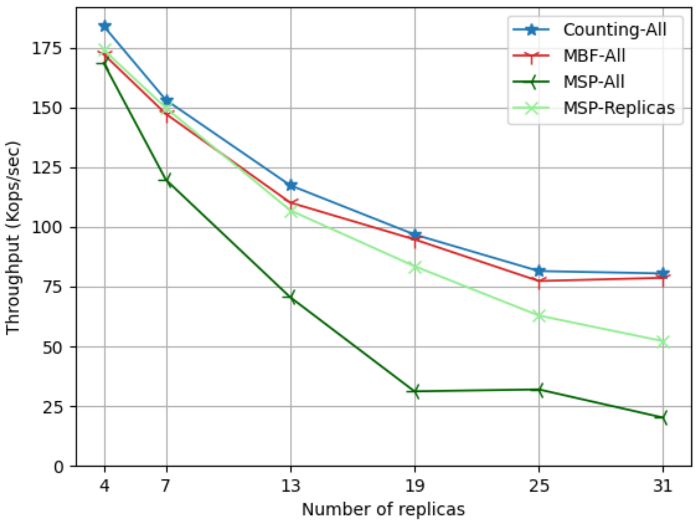
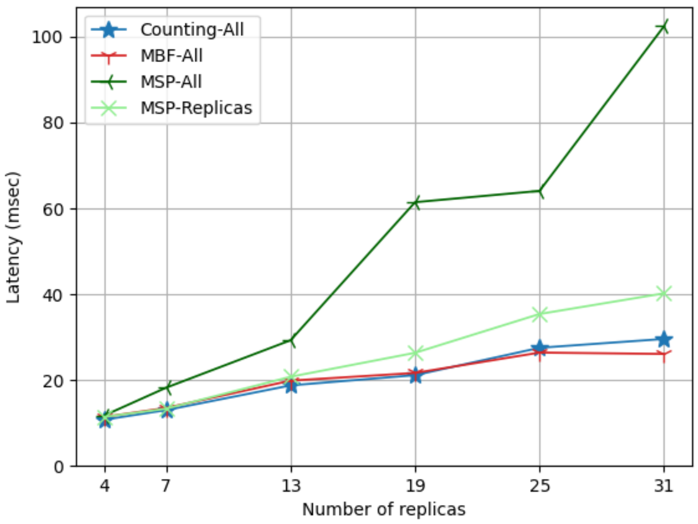
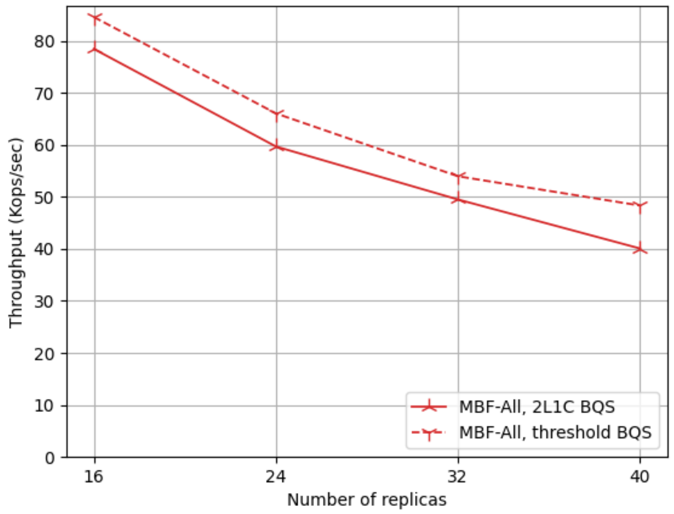
The MSP-Replicas protocol is still comparable to Counting-All

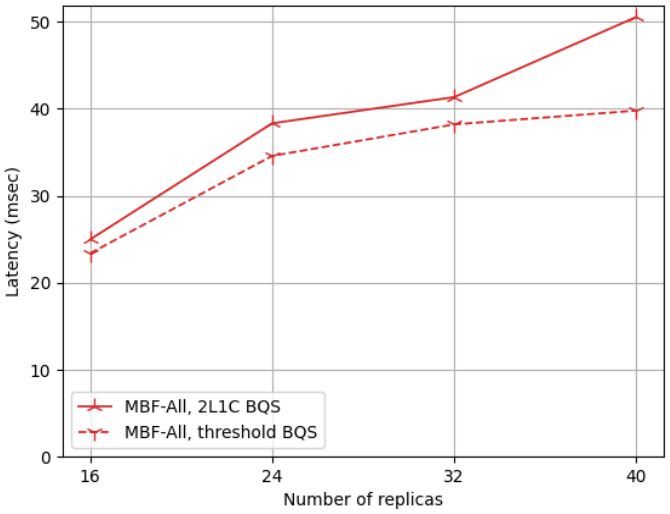




- Up to 31 replicas and 32 clients
- All systems again instantiated with a threshold BQS with $n = 3f + 1$

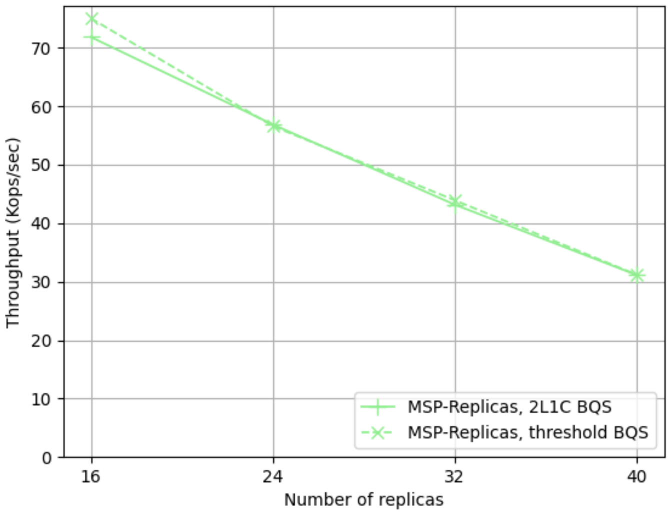
The complexity of the BQS moderately affects the MBF-All implementation

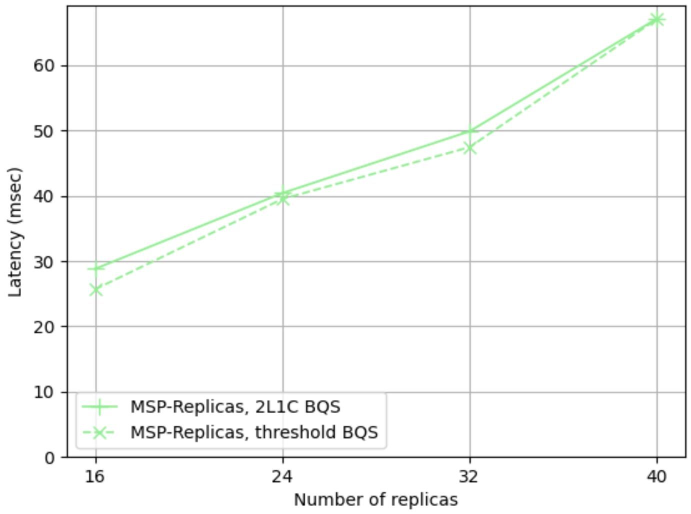




- 16 up to 40 replicas and 32 clients
- MBF-All instantiated with the threshold and the 2L1C BQS

The complexity of the BQS affects the MSP-Replicas protocol only slightly





- 16 up to 40 replicas and 32 clients
- MSP-Replicas instantiated with the threshold and the 2L1C BQS

The MBF-All protocol outperforms the MSP-Replicas

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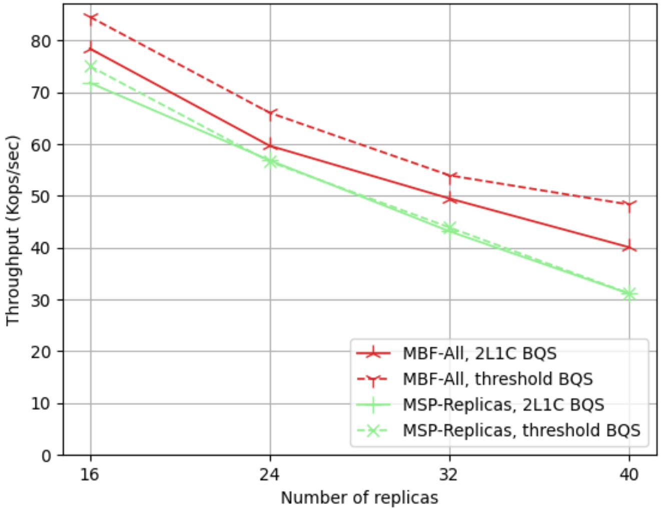

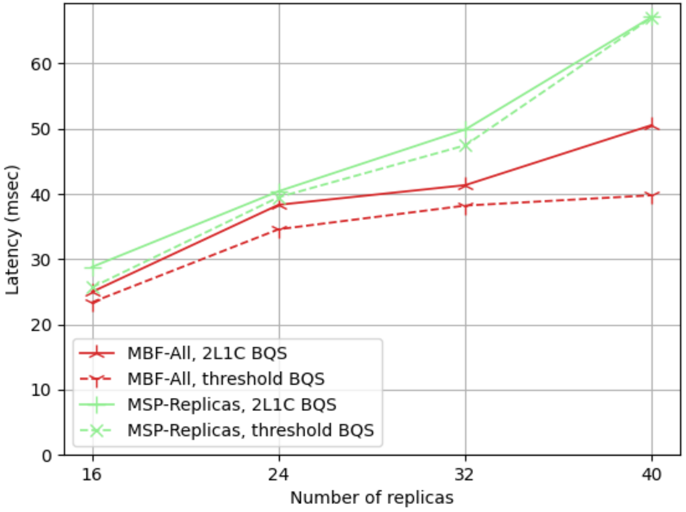

- 16 up to 40 replicas and 32 clients
- Systems instantiated with the threshold and the 2L1C BQS

Thank you!

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Full paper: arxiv.org/abs/2006.04616

Blogpost: cryptobern.github.io/beyondthreshold/

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